Impact of amplifying media on the Casimir force

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On the basis of macroscopic quantum electrodynamics, a theory of Casimir forces in the presence of linearly amplifying bodies is presented which provides a consistent framework for studying the effect of, e.g., amplifying left-handed metamaterials on dispersion forces. It is shown that the force can be given in terms of the classical Green tensor and that it can be decomposed into a resonant component associated with emission processes and an off-resonant Lifshitz-type component. We explicitly demonstrate that our theory extends additive approaches beyond the dilute-gas limit.

I. INTRODUCTION

Among the vast body of literature related to the Casimir forces [1–4] there is an almost complete lack of studies concerned with the influence of amplifying media. This is in stark contrast to the fact that Casimir forces between amplifying bodies may lead to far-reaching applications. To name just two examples, amplifying bodies may hold the key to enhance the impact of novel (meta)material properties on dispersion forces and to realizing repulsive Casimir forces.

The problem of dispersion forces in the presence of metamaterials has recently attracted a lot of attention [5–11]. However, passive metamaterials suffer from high absorption which restricts desired metamaterial properties such as lefthandedness [12, 13] to a narrow spectral bandwidth [14]; this may reduce or completely inhibit an influence of such properties on dispersion forces. It has been suggested that the influence of absorption can be mitigated via introducing active media [14]. In practice, gain may be introduced in a medium by optical parametric pumping [15] or quantum cascade lasing techniques [16, 17]. The potential of amplifying bodies as a means of realizing repulsive Casimir forces has been pointed out recently [18, 19]. It is already manifest in the Casimir–Polder (CP) forces acting on individual excited atoms [10, 20–22]. Repulsive dispersion forces can help to suppress the unwanted phenomenon of stiction and allows for new classes of nanodevices [23]. Note that the repulsive Casimir forces recently measured in Refs. [24, 25] require the interacting bodies to be embedded in medium. This is not necessary when realizing repulsive forces on the basis of amplification.

A recent calculation of Casimir forces on amplifying bodies [19] was based on the assumption that the well-known Lifshitz-type formula for the Casimir force as an integral over imaginary frequencies applies without change to amplifying media — an approach which neglects excited-state emission processes typical of amplification. As an alternative, the Casimir force between two dilute samples of excited atoms has been studied microscopically by summing over two-atom interactions [18]. Such an approach is limited to sufficiently dilute media, whereas a theory nonlinear in the magnetoelectric properties is indispensable in many applications such as superlens-scenarios [26] or anti-stiction tools [27].

In this Rapid Communication, we present a macroscopic nonperturbative theory of Casimir forces on amplifying bodies that presents a consistent generalisation of additive descriptions based on dispersion forces on atoms. We establish the consistency of our macroscopic body–body force with the theoretically [20, 21] and experimentally [22] well understood force between an excited atom and a body and show that relevant processes such as emission by the bodies are fully taken into account. We begin with an outline of the underlying quantisation scheme [28], which is an extension of macroscopic quantum electrodynamics (QED) to amplifying media, and then derive the Casimir force on an amplifying, polarisable body of arbitrary shape. Finally, contact to CP forces is established by applying the general formula to the case of a weakly polarisable medium.

II. QUANTISATION SCHEME

Consider an arrangement of polarisable bodies whose linear, local and isotropic response is described by a spatially varying complex permittivity $\varepsilon(\mathbf{r}, \omega)$ that fulfils the Kramers–Kronig relations. We allow for bodies that are (linearly) amplifying in some frequency range [$\mathrm{Im}\,\varepsilon(\mathbf{r}, \omega) = \varepsilon_I(\mathbf{r}, \omega) < 0$], assuming the amplifying medium to be pumped to a quasi-stationary excited state (for details, see [28]). Note that this inversion-type excitation is fundamentally different from thermal excitations. In particular, the amplifying bodies are explicitly

not at (thermal) equilibrium with their environment.

The quantised electric field can be given as the solution to the inhomogeneous Helmholtz equation

$$[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times -\omega^2/c^2\varepsilon(\mathbf{r},\omega)]\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) = i\mu_0\omega\underline{\hat{\mathbf{j}}}_N(\mathbf{r},\omega) \quad (1)$$

$[\hat{\mathbf{O}}(\mathbf{r}) = \int_0^\infty \mathrm{d}\omega\underline{\hat{\mathbf{Q}}}(\mathbf{r},\omega) + \text{H.c.}]$ according to

$$\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) = i\omega\mu_0 \int \mathrm{d}^3r'\, \mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_N(\mathbf{r}',\omega), \quad (2)$$

where the classical Green tensor obeys the equation

$$[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times -\omega^2/c^2]\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)$$
$$= \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}') + \omega^2/c^2[\varepsilon(\mathbf{r},\omega)-1]\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \quad (3)$$

together with the boundary condition at infinity. By exchanging the roles of creation and annihilation operators in the amplifying space and frequency regime [29], the noise current density can be given as

$$\underline{\hat{\mathbf{j}}}_N(\mathbf{r},\omega) = \omega\sqrt{\hbar\varepsilon_0\pi^{-1}|\varepsilon_I(\mathbf{r},\omega)|}$$
$$\times \left\{ \Theta[\varepsilon_I(\mathbf{r},\omega)]\hat{\mathbf{f}}(\mathbf{r},\omega) + \Theta[-\varepsilon_I(\mathbf{r},\omega)]\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega) \right\} \quad (4)$$

[Θ: unit step function, $\Theta(0) \equiv 1$]. The dynamical variables $\hat{\mathbf{f}}(\mathbf{r},\omega)$ as introduced in Eq. (4) obey bosonic commutation relations such that the equal-time commutation relation characteristic of the electromagnetic field holds, $[\hat{\mathbf{E}}(\mathbf{r}),\hat{\mathbf{B}}(\mathbf{r}')] = i\hbar\varepsilon_0^{-1}\boldsymbol{\nabla} \times \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$, where the electric and induction fields are given by Eq. (2) and

$$\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega) = \mu_0 \int \mathrm{d}^3r'\, \boldsymbol{\nabla} \times \mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_N(\mathbf{r}',\omega) \quad (5)$$

respectively. The vacuum state $|\{0\}\rangle$ of the electromagnetic field and the partially amplifying electric body is defined by $\hat{\mathbf{f}}(\mathbf{r},\omega)|\{0\}\rangle = \mathbf{0}\ \forall \mathbf{r},\omega$. The quantisation procedure implies that the Hamiltonian $\hat{H} = \hat{H}_+ + \hat{H}_- = \int \mathrm{d}^3r\int_0^\infty \mathrm{d}\omega\,\hbar\omega\{\Theta[\varepsilon_I(\mathbf{r},\omega)]-\Theta[-\varepsilon_I(\mathbf{r},\omega)]\}\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega)$ generates the correct Maxwell equations via the Heisenberg equations of motion.

III. CALCULATION OF THE CASIMIR FORCE

The Casimir force acting on a partially amplifying body of volume V in the presence of other bodies outside V can be identified as the average Lorentz force [30]

$$\mathbf{F} = \int_V \mathrm{d}^3r \langle\hat{\rho}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}') + \hat{\mathbf{j}}(\mathbf{r}) \times \hat{\mathbf{B}}(\mathbf{r}')\rangle_{\mathbf{r}'\to\mathbf{r}} \quad (6)$$

on the body's internal charge and current densities

$$\underline{\hat{\rho}}(\mathbf{r},\omega) = \frac{i\omega}{c^2}\boldsymbol{\nabla} \cdot \int \mathrm{d}^3r'\, \mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_N(\mathbf{r}',\omega), \quad (7)$$

$$\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega) = \left(\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times -\frac{\omega^2}{c^2}\right) \int \mathrm{d}^3r'\, \mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_N(\mathbf{r}',\omega). \quad (8)$$

Note that the coincidence limit $\mathbf{r}' \to \mathbf{r}$ has to be performed such that (divergent) self-forces are discarded.

On using Eq. (4), the bosonic commutation relations for $\hat{\mathbf{f}}$, $\hat{\mathbf{f}}^\dagger$, and the definition of the vacuum state, we find the following nonvanishing expectation values:

$$\langle\underline{\hat{\mathbf{j}}}_N(\mathbf{r},\omega)\underline{\hat{\mathbf{j}}}_N^\dagger(\mathbf{r}',\omega')\rangle = \hbar\omega^2\varepsilon_0\pi^{-1}\delta(\omega-\omega')\varepsilon_I(\mathbf{r},\omega)$$
$$\times \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')\Theta[\varepsilon_I(\mathbf{r},\omega)], \quad (9)$$

$$\langle\underline{\hat{\mathbf{j}}}_N^\dagger(\mathbf{r},\omega)\underline{\hat{\mathbf{j}}}_N(\mathbf{r}',\omega')\rangle = -\hbar\omega^2\varepsilon_0\pi^{-1}\delta(\omega-\omega')\varepsilon_I(\mathbf{r},\omega)$$
$$\times \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')\Theta[-\varepsilon_I(\mathbf{r},\omega)]. \quad (10)$$

We evaluate the force by combining Eqs. (2),(5), (6)–(10) and writing $\mathbf{a} \times \mathbf{b} = -\text{Tr}(\mathbf{I} \times \mathbf{a} \otimes \mathbf{b})$ ($[\text{Tr}\mathbf{T}]_i = T_{kik}$) for the $\hat{\mathbf{j}} \times \hat{\mathbf{B}}$ term. Eliminating $\Theta[\varepsilon_I(\mathbf{s},\omega)]$ according to $\Theta[\varepsilon_I(\mathbf{s},\omega)] = 1-\Theta[-\varepsilon_I(\mathbf{s},\omega)]$, those spatial integrals not depending on $\Theta[-\varepsilon_I(\mathbf{s},\omega)]$ can be performed via

$$\frac{\omega^2}{c^2} \int \mathrm{d}^3s\,\varepsilon_I(\mathbf{s},\omega)\mathbf{G}(\mathbf{r},\mathbf{s},\omega) \cdot \mathbf{G}^*(\mathbf{s},\mathbf{r}',\omega) = \text{Im}\,\mathbf{G}(\mathbf{r},\mathbf{r}',\omega),$$
$$(11)$$

so the Casimir force can be given by $\mathbf{F} = \mathbf{F}^\mathrm{r} + \mathbf{F}^\mathrm{nr}$ with

$$\mathbf{F}^\mathrm{nr} = \frac{\hbar}{\pi} \int_0^\infty \mathrm{d}\omega \int_V \mathrm{d}^3r \left\{ \frac{\omega^2}{c^2}\boldsymbol{\nabla} \cdot \text{Im}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \right.$$
$$\left. + \text{Tr}\left[\mathbf{I}\times\left(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times -\frac{\omega^2}{c^2}\right)\text{Im}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\times\overleftarrow{\boldsymbol{\nabla}}'\right] \right\}_{\mathbf{r}'\to\mathbf{r}}$$
$$(12)$$

and

$$\mathbf{F}^\mathrm{r} = -\frac{2\hbar}{\pi c^2} \int_V \mathrm{d}^3r \int_0^\infty \mathrm{d}\omega\omega^2 \int \mathrm{d}^3s\varepsilon_I(\mathbf{s},\omega)\Theta[-\varepsilon_I(\mathbf{s},\omega)]$$
$$\times \text{Re}\left\{\omega^2/c^2\boldsymbol{\nabla} \cdot \mathbf{G}(\mathbf{r},\mathbf{s},\omega) \cdot \mathbf{G}^*(\mathbf{s},\mathbf{r}',\omega)\right.$$
$$+ \text{Tr}[\mathbf{I} \times (\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times -\omega^2/c^2)\mathbf{G}(\mathbf{r},\mathbf{s},\omega)$$
$$\left. \cdot \mathbf{G}^*(\mathbf{s},\mathbf{r}',\omega) \times \overleftarrow{\boldsymbol{\nabla}}']\right\}_{\mathbf{r}'\to\mathbf{r}}. \quad (13)$$

Equations (12) and (13) represent general expressions for the Casimir force acting on a linearly polarisable body of arbitrary shape and material in an arbitrary environment of additional bodies or media, where any of the bodies may be amplifying. The term $\mathbf{F}^\mathrm{nr}$ is a purely nonresonant Lifshitz-type contribution to the force. It can be rewritten as an integral over purely imaginary frequencies $\omega = i\xi$ and has exactly the same form as for purely absorbing bodies. In Ref. [19], the nonresonant term $\mathbf{F}^\mathrm{nr}$ is identified with the total Casimir force and it is shown that $\mathbf{F}^\mathrm{nr}$ may become repulsive in the presence of amplifying media as a consequence of the property $\varepsilon(i\xi) \leq 1$. The resonant term $\mathbf{F}^\mathrm{r}$ has never been given before. It only arises in the presence of amplifying bodies, in which case it can dominate the total Casimir force. As evident from the factor $\Theta[-\varepsilon_I(\mathbf{s},\omega)]$, the force component $\mathbf{F}^\mathrm{r}$ is associated with emission processes [the emission spectrum being related to $-\varepsilon_I(\mathbf{s},\omega)$].

IV. CONTACT TO CASIMIR–POLDER FORCES

We will next establish a relation between the Casimir force $\mathbf{F} = \mathbf{F}^{\mathrm{r}} + \mathbf{F}^{\mathrm{nr}}$ [with $\mathbf{F}^{\mathrm{r}}$ and $\mathbf{F}^{\mathrm{nr}}$ being given by Eqs. (12) and (13), respectively] and the well-understood CP force on excited atoms. In this way, we will be able to substantiate the role of emission mentioned above. To that end, we consider the Casimir force on an optically dilute amplifying body of volume V placed in a free-space region in an environment of purely absorbing bodies. We follow the procedure outlined in Ref. [30] for an absorbing dielectric body.

We begin with the nonresonant force component $\mathbf{F}^{\mathrm{nr}}$, Eq. (12), and explicitly introduce the electric susceptibility $\chi(\mathbf{r}, \omega) = \varepsilon(\mathbf{r}, \omega) - 1$ ($\mathbf{r} \in V$) of the body by invoking the relations

$$\left(\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times - \frac{\omega^2}{c^2} \right) \mathrm{Im}\, \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \frac{\omega^2}{c^2} \, \mathrm{Im}[\chi(\mathbf{r}, \omega) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)],$$
(14)

$$\boldsymbol{\nabla} \cdot \mathrm{Im}\, \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = -\mathrm{Im}[\boldsymbol{\nabla} \cdot \chi(\mathbf{r}, \omega) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)], \quad (15)$$

which follow from Eq. (3). Expanding the resulting expression for $\mathbf{F}^{\mathrm{nr}}$ via $\mathrm{Tr}[\mathbf{I} \times \mathbf{G} \times \overleftarrow{\boldsymbol{\nabla}}'] = \boldsymbol{\nabla}'\mathrm{Tr}\mathbf{G} - \boldsymbol{\nabla}' \cdot \mathbf{G}$ and exploiting the fact that terms involving a total divergence can be converted to a surface integral that vanishes for a body in free space, one obtains

$$\mathbf{F}^{\mathrm{nr}} = \frac{\hbar}{2\pi} \int_V \mathrm{d}^3 r \int_0^\infty \mathrm{d}\omega \, \frac{\omega^2}{c^2}$$
$$\times \mathrm{Im}\left[\chi(\mathbf{r}, \omega) \boldsymbol{\nabla} \mathrm{Tr}\mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}, \omega)\right], \quad (16)$$

where the symmetry $\mathbf{G}(\mathbf{r}', \mathbf{r}, \omega) = \mathbf{G}^{\mathrm{T}}(\mathbf{r}, \mathbf{r}', \omega)$ of the Green tensor has been used. In addition, we have assumed the body to be homogeneous and performed the coincidence limit by simply replacing the Green tensor with its scattering part $\mathbf{G}^{(1)}$ (see the discussion in Ref. [30]). Next, we exploit the fact that the amplifying body is optically dilute and expand $\mathbf{F}^{\mathrm{nr}}$ as given by Eq. (16) to leading (linear) order in the susceptibility $\chi(\mathbf{r}, \omega)$ of the amplifying body. We thus have to replace $\mathbf{G}$ with its zero-order approximation $\overline{\mathbf{G}}$, i.e., the Green tensor of the system in the absence of the amplifying body which is the solution to the Helmholtz equation (3) with

$$\overline{\varepsilon}(\mathbf{r}, \omega) = \begin{cases} \varepsilon(\mathbf{r}, \omega) & \text{for } \mathbf{r} \notin V, \\ 1 & \text{for } \mathbf{r} \in V. \end{cases} \quad (17)$$

in place of $\varepsilon(\mathbf{r}, \omega)$. Finally, we assume that the amplifying body consists of a gas of isotropic atoms in an excited state $|n\rangle$ with polarizability

$$\alpha_n(\omega) = \lim_{\epsilon \to 0} \frac{1}{3\hbar} \sum_k \left[\frac{|\mathbf{d}_{nk}|^2}{\omega + \omega_{kn} + i\epsilon} - \frac{|\mathbf{d}_{nk}|^2}{\omega - \omega_{kn} + i\epsilon} \right] \quad (18)$$

(ω_{kn}: transition frequencies, $\mathbf{d}_{nk}$: electric dipole matrix elements), which can be related to the electric susceptibility of the body via the linearised Clausius–Mossotti law

$\chi(\omega) = \varepsilon_0^{-1} \eta \alpha_n(\omega)$ (η: atomic number density). Transforming the frequency integral to the positive imaginary axis, we obtain $\mathbf{F}^{\mathrm{nr}} = -\int \mathrm{d}^3 r \, \eta \boldsymbol{\nabla} U_n^{\mathrm{nr}}(\mathbf{r})$, where

$$U_n^{\mathrm{nr}}(\mathbf{r}) = \frac{\hbar\mu_0}{2\pi} \int_0^\infty \mathrm{d}\xi \, \xi^2 \alpha_n(i\xi) \mathrm{Tr}\overline{\mathbf{G}}^{(1)}(\mathbf{r}, \mathbf{r}, i\xi) \quad (19)$$

is the nonresonant CP potential of the excited body atoms [21]. It should be pointed out that there is an important difference to the case of the force on an absorbing object made of ground-state atoms: In the latter case, all of the frequencies ω_{kn} in Eqs. (18) are positive so that the respective (virtual) transitions contribute to the nonresonant CP potential with the same sign. For excited atoms, upward as well as downward transitions are possible, so that positive and negative ω_{kn} occur and the overall sign of the nonresonant force can be reversed to make it repulsive.

Let us next consider the resonant force component $\mathbf{F}^{\mathrm{r}}$, Eq. (13), following similar steps as above. The linear approximation of $\mathbf{F}^{\mathrm{r}}$ can be obtained by using the zero-order approximation to Eq. (3) together with the identity $\omega^2/c^2 \boldsymbol{\nabla} \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = -\boldsymbol{\nabla}\delta(\mathbf{r} - \mathbf{r}')$ and replacing $\mathbf{G}^*$ with $\overline{\mathbf{G}}^*$. Expanding the result according to $\mathrm{Tr}[\mathbf{I} \times \mathbf{G} \times \overleftarrow{\boldsymbol{\nabla}}'] = \boldsymbol{\nabla}'\mathrm{Tr}\mathbf{G} - \boldsymbol{\nabla}' \cdot \mathbf{G}$ and discarding, for a body in free space, terms involving total divergences we derive

$$\mathbf{F}^{\mathrm{r}} = -\frac{\hbar}{\pi} \int_V \mathrm{d}^3 r \int_0^\infty \mathrm{d}\omega \, \frac{\omega^2}{c^2} \, \Theta[-\varepsilon_I(\mathbf{r}, \omega)]\varepsilon_I(\mathbf{r}, \omega)$$
$$\times \boldsymbol{\nabla}\mathrm{Tr}\,\mathrm{Re}\,\overline{\mathbf{G}}^{(1)}(\mathbf{r}, \mathbf{r}, \omega), \quad (20)$$

where we have again assumed the body to be homogeneous and performed the coincidence limit by replacing the Green tensor with its scattering part. Relating ε_I to the polarizability of the atoms by means of the linearised Clausius–Mossotti relation, we finally obtain $\mathbf{F}^{\mathrm{r}} = -\int \mathrm{d}^3 r \, \eta \boldsymbol{\nabla} U_n^{\mathrm{r}}(\mathbf{r})$, where

$$U_n^{\mathrm{r}}(\mathbf{r}) = \frac{\hbar\mu_0}{\pi} \int_0^\infty \mathrm{d}\omega \, \omega^2 \Theta[-\mathrm{Im}\,\alpha_n(\omega)]\mathrm{Im}\,\alpha_n(\omega)$$
$$\times \mathrm{Tr}\,\mathrm{Re}\,\overline{\mathbf{G}}^{(1)}(\mathbf{r}, \mathbf{r}, \omega) \quad (21)$$

is nothing but the resonant part of the CP potential of the excited atoms contained in the body. By using the relation

$$\alpha_I(\omega) = \frac{\pi}{3\hbar} \sum_k |\mathbf{d}_{nk}|^2 [\delta(\omega + \omega_{nk}) - \delta(\omega - \omega_{nk})], \quad (22)$$

which follow from definition (18) together with the identity $\lim_{\epsilon \to 0} 1/(x + i\epsilon) = \mathcal{P}/x - i\pi\delta(x)$ ($\mathcal{P}$: principal value), we can write it in the more familiar form [21]

$$U_n^{\mathrm{r}}(\mathbf{r}) = -\frac{\mu_0}{3} \sum_k \Theta(\omega_{nk})\omega_{nk}^2 |\mathbf{d}_{nk}|^2 \mathrm{Tr}\mathrm{Re}\overline{\mathbf{G}}^{(1)}(\mathbf{r}, \mathbf{r}, \omega_{nk}).$$
(23)

Combining the results for $\mathbf{F}^{\mathrm{nr}}$ and $\mathbf{F}^{\mathrm{r}}$, we see that the Casimir force on an optically dilute, homogeneous, amplifying electric body is the sum of the CP forces on the

excited atoms contained therein,

$$\mathbf{F} = \mathbf{F}^{\mathrm{nr}} + \mathbf{F}^{\mathrm{r}} = -\int \mathrm{d}^3 r \eta \boldsymbol{\nabla} U_n(\mathbf{r}) \qquad (24)$$

$[U_n(\mathbf{r}) = U_n^{\mathrm{nr}}(\mathbf{r}) + U_n^{\mathrm{r}}(\mathbf{r})]$. This result generalises similar findings for purely absorbing bodies (consisting of ground-state atoms) [30–33] to amplifying ones. In particular, the nonresonant and resonant components of the Casimir force, Eqs. (12) and (13), are directly related to the respective CP-potential terms which in turn are associated with virtual and real transitions of the atoms. Recall that for an atom in front of a plate at zero temperature, the nonresonant CP potential is proportional to $1/z^3$ and $1/z^4$ in the nonretarded and retarded limits (z: atom–plate separation), while the resonant potential as governed by a $1/z^3$ power law in the nonretarded regime makes way for a spatially oscillating $\cos(2\omega_{nk}z/c)/z$ asymptote for retarded distances [20]. As we have seen, the most important difference between forces on amplifying as opposed to absorbing bodies is the presence of a strong, resonant force contribution which is associated with real transitions of the excited body atoms and hence with emission processes of the body.

V. SUMMARY AND PERSPECTIVE

On the basis of the consistent framework provided by macroscopic QED, we have developed an exact theory of Casimir forces in arrangements of linearly responding, electrically polarisable bodies of arbitrary shape, with special emphasis on amplifying bodies. The formulas (12) and (13) show that the Casimir force can be decomposed into two parts: A nonresonant Lifshitz-type component that looks formally the same as in the case of a purely absorbing body and a novel, resonant component that is a direct consequence of the amplification in the system and contains an integration over bodies and frequencies where the imaginary part of the electric permittivity is negative. We have demonstrated that in the dilute-gas limit, the Casimir force on an amplifying body in the presence of absorbing bodies is given by a sum of CP forces over the excited body atoms; our theory is hence the natural generalisation of additive approaches beyond their scope.

Two points to be addressed in more detail in the future are (i) the finite-temperature case and (ii) the relation of the excited atom–body force to atom–atom forces. (i) Finite temperature of the absorbing environment may be accounted for by introducing a thermal density matrix $\hat{\rho}_T = \exp[-\hat{H}_+/(k_{\mathrm{B}}T)]/\mathrm{Tr}\{\exp[-\hat{H}_+/(k_{\mathrm{B}}T)]\}$. This leads to a factor $[2n_T(\omega)+1]$ with thermal photon number $n_T(\omega)$ in Eq. (12), so the ξ-integral in Eq. (19) will be replaced with a Matsubara sum, cf. Ref. [34]. The $T \to 0$ limit of the nonresonant force being a nonuniform asymptotic expansion [35, 36], the $T = 0$ results are only valid for distances $z \ll \hbar c/(2\pi k_{\mathrm{B}}T)$. (ii) The theoretically and experimentally established resonant and spatially oscillating forces between excited atoms and bodies are in contrast with the non-oscillating atom–atom forces found in the majority of theoretical works [37, 38]. The discrepancy indicates that either the inclusion of atomic linewidth at the heart of excited atom–atom calculations has to be carefully reconsidered [39] or that microscopic atom–atom interactions are fundamentally different from the collective atom–body interaction.

Our results, which can be extended to magnetoelectric, anisotropic or nonlocally responding media in a straightforward way, present a reliable framework for enhancing the impact of metamaterial properties such as negative refraction on dispersion forces or realizing repulsive forces. In particular, perfect lens scenarios can be investigated by using the appropriate Green tensors. Note that the strategy employed in this Rapid Communication can also be employed to study CP forces on atoms in the presence of amplifying bodies, where a rich dynamics associated with the exchange of excitations is to be expected.

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